Simulation Modeling

Goal: Use probabilistic methods to analyze deterministic and probabilistic models.

Example. Determine the best elevator delivery scheme.

- The wait is too long, too many stops along the way.
- Inconvenient to experiment with alternate delivery schemes.
  - Disrupt normal service
  - Take surveys of customers
  - Confuse regular customers
- Alternatively, run a computer **simulation**. Write a computer program that models the system of elevators, including:
  - Time of arrival of passengers (a random event)
  - Passenger destination (a random event)
  - Capacity of elevator (fixed by system)
  - Speed of elevator (fixed by system)
  - Current delivery scheme

Simulation Modeling

Once you have written the computer program,

Verify that the simulation models the current real-world situation

- Run the model many times.
- Have the computer keep track of data, such as average wait time, number of stops it takes, longest queue, etc.

Then, modify various parameters in order to simulate a new delivery scheme.

- How do the data change?
- Is the alternate scheme better or worse?
- Determine how to implement to cause minimal disruption.

Monte Carlo Simulations

Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

PROS:

- It is a relatively easy method to approximate complex systems.
- Once built, it allows for tinkering—easy to do sensitivity analysis.
- It can model systems over difficult-to-measure time frames.

CONS:

- You have to build it. (Expensive to develop!)
- Requires computing power and time.
- Makes you over-confident in the results.
- Dealing with probability, so results will always be of the form: "With 95% probability, the wait time will be less than 2 minutes."

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

To simulate a random event, use one of the *Mathematica* commands:

- `RandomInteger` gives a pseudo-random *integer*.
  - `RandomInteger[]` (no input) gives either 0 or 1.
  - `RandomInteger[5]` gives an integer from 0 to 5.
  - `RandomInteger[{1, 10}]` gives an integer from 1 to 10.
  - `RandomInteger[{1, 10},20]` gives a list of 20 such integers.
- `RandomReal` gives a pseudo-random *real number*.
  - `RandomReal[]` (no input) gives a real number between 0 or 1.
  - `RandomReal[{0.1, 0.2}]` gives a real number from 0.1 to 0.2.
  - `RandomReal[{0.1, 0.2},15]` gives a list of 15 such numbers.

The first input gives the range; a second input tells how many to make.

The numbers produced by a random number generator are never truly random because they are produced by an algorithm on a deterministic machine.

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

Let's use the convention: 1=‘Head’ and 0=‘Tail’. Then evaluating `RandomInteger[1,20]` will generate a list of 20 coin tosses.

In[1]: `CoinFlips = RandomInteger[1,20]`
Out[1]: $\{1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1\}$

The sum of this list is the total number of heads tossed.

In[2]: `Total[CoinFlips]`
Out[2]: 13

Running the commands again will simulate another trial of 20 flips.

If statements and For loops

In order to incorporate more complex aspects into the model, it will be helpful to use both `If` statements and `For` loops.

- `If[condition,t,f]` checks the 'condition'. If 'condition' is true, the statement evaluates 't'. Otherwise, it evaluates 'f'.
  - The command '`If[x<0, -x, x]`' compares x with 0. If x is less than zero, the output is $-x$. Otherwise, the output is x itself.
- `For[start,test,incr,body]` evaluates 'start', and continues to evaluate 'body' and increment 'incr' until 'test' is false.
  - `For[i = 0, i < 4, i++, Print[i]]` first starts by setting i to 0. It then checks to see if i is less than 4. It is, so the command evaluates '`Print[i]`', and increments i by 1 (`i++`). Now $i = 1$, which is still < 4, so '`Print[i]`' is evaluated and i is incremented. Similarly for $i = 2$ and $i = 3$. Now i is incremented to 4, which is NOT < 4, and the loop terminates.

Be careful to name counters wisely!

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

Think about how we are going to set up a `for` loop:

Pseudocode: (won't actually work if we type into a computer)

- For i from 1 to 20,
  - Generate a random integer between 0 and 1.
  - If '1' output 'Head', if '0', output 'Tail'.

```
For[i = 1, i <= 20, i++,
   If[RandomInteger[]==1,Print["Head"],Print["Tail"]]]
```

- Notice the '==' in the `If` statement, needed for comparison.
- i simply serves as a counter, not used at each step's evaluation.

Simulating flipping a coin

Pimp my code! Let's keep track of how many heads and tails are thrown by introducing other counters.

- Reset the counters: 'headCount=0' and 'tailCount=0'.
- For i from 1 to 20,
  - Generate a random integer between 0 and 1.
  - If '1' output 'Head' and increase 'headCount', if '0', output 'Tail' and increase 'tailCount'.
- Display 'headCount' and 'tailCount'.

```
headCount=0; tailCount=0;
For[i = 1, i <= 20, i++, If[RandomInteger[]==1,
 Print["Head"]; headCount++, Print["Tail"],tailCount++]]
{headCount, tailCount}
```

- Sample output: Head, Tail, Tail, etc. $\{12, 8\}$
- Note the semicolon between successive commands in the parts of the `if` statement.

Simulating rolling a biased die

Suppose you have a four-sided die, where the four sides (A, B, C, and D) come up with probabilities 1/2, 1/4, 1/8, and 1/8, respectively.

- Reset the counters: 'aCount=bCount=cCount=dCount=0'.
- For *i* from 1 to 20,
  - Generate a random real number between 0 and 1.
  - If between 0 and 1/2, then output 'A' and aCount++
    if between 1/2 and 3/4, then output 'B' and bCount++
    if between 3/4 and 7/8, then output 'C' and cCount++
    if between 7/8 and 1, then output 'D' and dCount++
- Display 'aCount', 'bCount', 'cCount', and 'dCount'.

Simulating rolling a biased die

```
aCount = 0; bCount = 0; cCount = 0; dCount = 0;
For[i = 1, i <= 20, i++, roll=RandomInteger[];
 If[0 <= roll < 1/2, Print["a"]; aCount++];
 If[1/2 <= roll < 3/4, Print["b"]; bCount++];
 If[3/4 <= roll < 7/8, Print["c"]; cCount++];
 If[7/8 <= roll <= 1, Print["d"]; dCount++];]
{aCount, bCount, cCount, dCount}
```

- Sample output:
  a, a, a, d, d, b, a, a, d, a, a, a, a, d, b, a, a, c, a, b $\{12, 3, 1, 4\}$
- These `If` statements all have no "False" part. (; vs ,)
- If you are feeling fancy, you can use one `Which` command instead of four `If` commands.
- *Important:* You MUST set a variable for the roll. Otherwise, calling `RandomInteger` four times will have you comparing different random numbers in each `If` statement.

Using Simulation to Calculate Area

Suppose you have a region whose area you don't know. You can approximate the area using a Monte Carlo simulation.

Idea: Surround the region by a rectangle. Randomly chosen points in the rectangle will fall in the region with probability

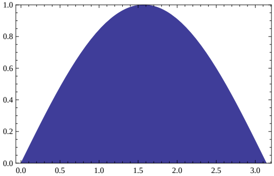
(area of region)/(area of rectangle)

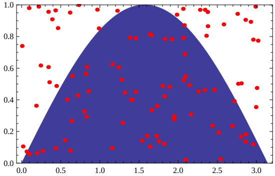
We can approximate this probability by calculating

(points falling in region)/(total points chosen).

Using Simulation to Calculate Area

Example. What is the area under the curve $\sin(x)$ from 0 to π?





Randomly select 100 points from the rectangle $[0, \pi] \times [0, 1]$.

[*Choose a random real between* 0 *and* π *for the x-coordinate and a random real between* 0 *and* 1 *for the y-coordinate*. . .]

$$\text{Then, } \frac{\text{Area of region}}{\underline{\hspace{2cm}}} \approx \frac{\text{Number of points in region}}{100}.$$

Here, 63 points fell in the region; we estimate the area to be ______.

Compare this to the actual value, $\int_{x=0}^{x=\pi} \sin(x)\,dx = [-\cos(x)]_{x=0}^{x=\pi} = 2$